Replication Bayes Factor for Correlations

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A replication Bayes factor (Verhagen and Wagenmakers, submitted) answers the question: "Is the effect from the replication attempt comparable to what was found before, or is it absent?". When a correlational study is replicated, the replication Bayes factor compares evidence in favor of the null hypothesis of no effect, $\mathcal{H}_0 : \rho = 0$, with the evidence in favor of the alternative hypothesis that the effect is equal to the effect found in the original study , $\mathcal{H}_r : \rho \sim$ "posterior distribution from original study".

The replication Bayes factor is computed in two steps:

1. In the first step the posterior distribution of the original study is obtained, assuming a uniform prior on the correlation. The density of this posterior distribution was given by Jeffreys (1961, pp 175, equation 9), and simplifies to:

$$p(\rho \mid Y_{orig}) = \frac{\frac{(1-\rho^2)^{\frac{1}{2}n}}{(1-\rho r)^{n-\frac{1}{2}}}\sqrt{\frac{\pi}{2}}\frac{\Gamma(n)}{\Gamma(n+\frac{1}{2})}\ {}_2F_1(\frac{1}{2},\frac{1}{2},n+\frac{1}{2},\frac{1}{2}+\frac{1}{2}r\rho)}{\int \frac{(1-\rho^2)^{\frac{1}{2}n}}{(1-\rho r)^{n-\frac{1}{2}}}\sqrt{\frac{\pi}{2}}\frac{\Gamma(n)}{\Gamma(n+\frac{1}{2})}\ {}_2F_1(\frac{1}{2},\frac{1}{2},n+\frac{1}{2},\frac{1}{2}+\frac{1}{2}r\rho)d\rho} \quad (1)$$

where ${}_2F_1$ is Gauss' hypergeometric function (Abramowitz and Stegun 1970, sec. 15).

2. The second step consists of the computation of the following equation integrated over the posterior distribution obtained in step 1:

$$\int \frac{(1-\rho^2)^{\frac{n}{2}}}{(1-\rho r)^{(n-\frac{1}{2})}} p(\rho \mid Y_{orig}) d\rho \quad (2)$$

$$\begin{aligned} \mathrm{B}_{10} &= \frac{p(Y \mid \mathcal{H}_1)}{p(Y \mid \mathcal{H}_0)} \\ &= \frac{\int p(Y \mid \delta, \mathcal{H}_1) p(\delta \mid \mathcal{H}_1)\, \mathrm{d}\delta}{p(Y \mid \mathcal{H}_0)} \\ &= \frac{\int \frac{(1-\rho^2)^{\frac{n}{2}}}{(1-\rho r)^{(n-\frac{1}{2})}} p(\rho \mid Y_{orig}) \mathrm{d}\rho}{p(\rho = 0 \mid Y_{orig})} \\ &= \int \frac{(1-\rho^2)^{\frac{n}{2}}}{(1-\rho r)^{(n-\frac{1}{2})}} p(\rho \mid Y_{orig}) \mathrm{d}\rho \end{aligned} \quad (3)$$

which can be done by performing a one-dimensional integration.

R code to perform this analysis can be found in this link `http://www.josineverhagen.com/?page_id=76`.