Replication Bayes Factor for Correlations

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A replication Bayes factor (Verhagen and Wagenmakers, submitted) answers the question: “Is the effect from the replication attempt comparable to what was found before, or is it absent?”. When a correlational study is replicated, the replication Bayes factor compares evidence in favor of the null hypothesis of no effect, $H_0: \rho = 0$, with the evidence in favor of the alternative hypothesis that the effect is equal to the effect found in the original study, $H_1: \rho \sim \text{posterior distribution from original study}$.

The replication Bayes factor is computed in two steps:

1. In the first step the posterior distribution of the original study is obtained, assuming a uniform prior on the correlation. The density of this posterior distribution was given by Jeffreys (1961, pp 175, equation 9), and simplifies to:

$$p(\rho \mid Y_{orig}) = \frac{(1-\rho^2)^\frac{n}{2}}{(1-\rho r)^{(n-1)/2}} \frac{\Gamma(n)}{\Gamma\left(\frac{n+1}{2}\right)} 2F_1\left(\frac{1}{2}, \frac{1}{2}, n + \frac{1}{2}, \frac{1}{2} + \frac{1}{2}r\rho\right)$$

where $2F_1$ is Gauss' hypergeometric function (Abramowitz and Stegun 1970, sec. 15).

2. The second step consists of the computation of the following equation integrated over the posterior distribution obtained in step 1:

$$B_{10} = \frac{p(Y \mid H_1)}{p(Y \mid H_0)} = \frac{\int p(Y \mid \delta, H_1)p(\delta \mid H_1) \, d\delta}{p(Y \mid H_0)} = \frac{\int \frac{(1-\rho^2)^\frac{n}{2}}{(1-\rho r)^{(n-1)/2}} p(\rho \mid Y_{orig}) \, d\rho}{p(\rho = 0 \mid Y_{orig})}$$

which can be done by performing a one-dimensional integration.
R code to perform this analysis can be found in this link http://www.josineverhagen.com/?page_id=76.